Vorlesung

Grundlagen der Künstlichen Intelligenz

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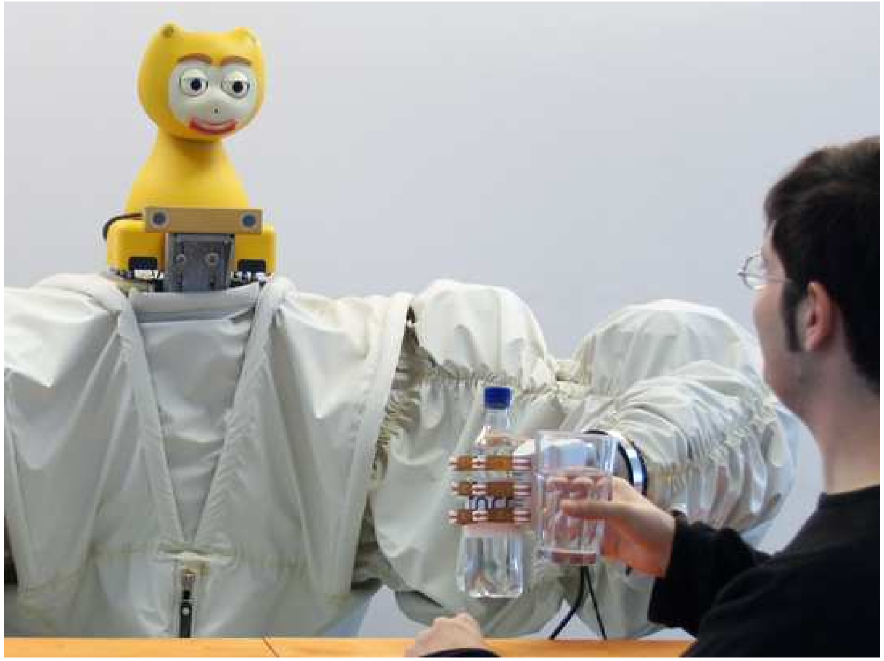

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Chapter 3

Solving Problems by Searching: Informed (Heuristic) Search

What’ the problem?

Combinatorial explosion:

- Uninformed search leads to exponential time and can only be solved for small problems
  - 15-puzzle: 10^{13} configurations
  - Rubik‘s cube: 4 x 10^{19} configurations
    - 1 million years with 1 turn per second
  - Chess: 10^{120} configurations (asuming ~ 40 moves)

How to solve it?

- Use additional information to reduce complexity
- Choose the node to expand based on an estimation on how fast the goal can be reached

Heuristics and their properties

Make use of domain knowledge:
„more knowledge, less search“

- Domain knowledge can be considered as „rules of thumb“
- Heuristics are simple rules that evaluate nodes with respect to the distance to the goal
- Good heuristics are
  - Good estimators
  - Simple and fast to compute

Best-first Search

- Information about the costs from a given node to the goal:
  - Evaluation function *h*, giving a real number for each node
  - Ideal case:
    - Knowing the correct costs from the node to the goal
  - Simple heuristics:
    - Euklidian distance
    - Manhatten distance

- Modify the generic graph-search algorithm using the heuristics

- When *h* is correct, i.e. estimation gives the actual costs: Follow the path of lowest cost, no need to search

Modify generic graph-search algorithm for best-first search

```
function HEURISTIC-SEARCH(problem, h)
returns  a solution or an error

static: open,  the initial state (set of nodes)
        closed, the nodes already visited, initally empty set

forever
        if open is empty then return error
        take a node out of open
        add this node to closed
        if this node contains a goal state then return solution
        expand this node (i.e. take all successors not in closed)
        add successor nodes to open using h
```

- Way of adding successor nodes defined by the heuristics

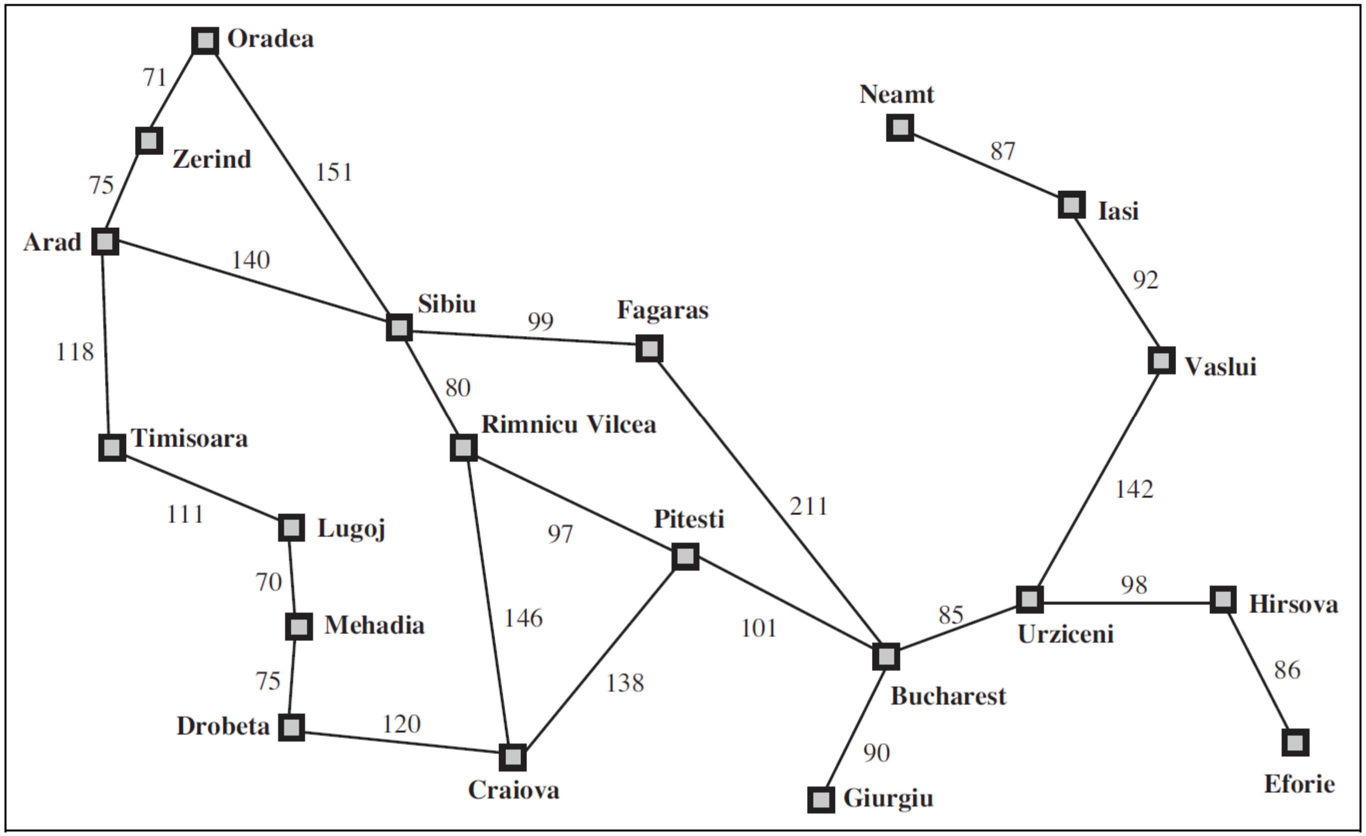
Greedy best-first search

- The „goodness“ of a node is determined by the distance to the goal

  $h(n)$ = estimated distance from node n to the goal
- Constraint for *h*: $h(n) = 0$, if n is a goal node

- In path planning: Direct distance between two locations

Greedy best-first search: From Arad to Bucharest



Air-line distances to Bucharest

Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Drobeta	242	Pitesti	100
Eforie	161	Rimnicu Vilcea	193
Fagaras	176	Sibiu	253
Giurgiu	77	Timisoara	329
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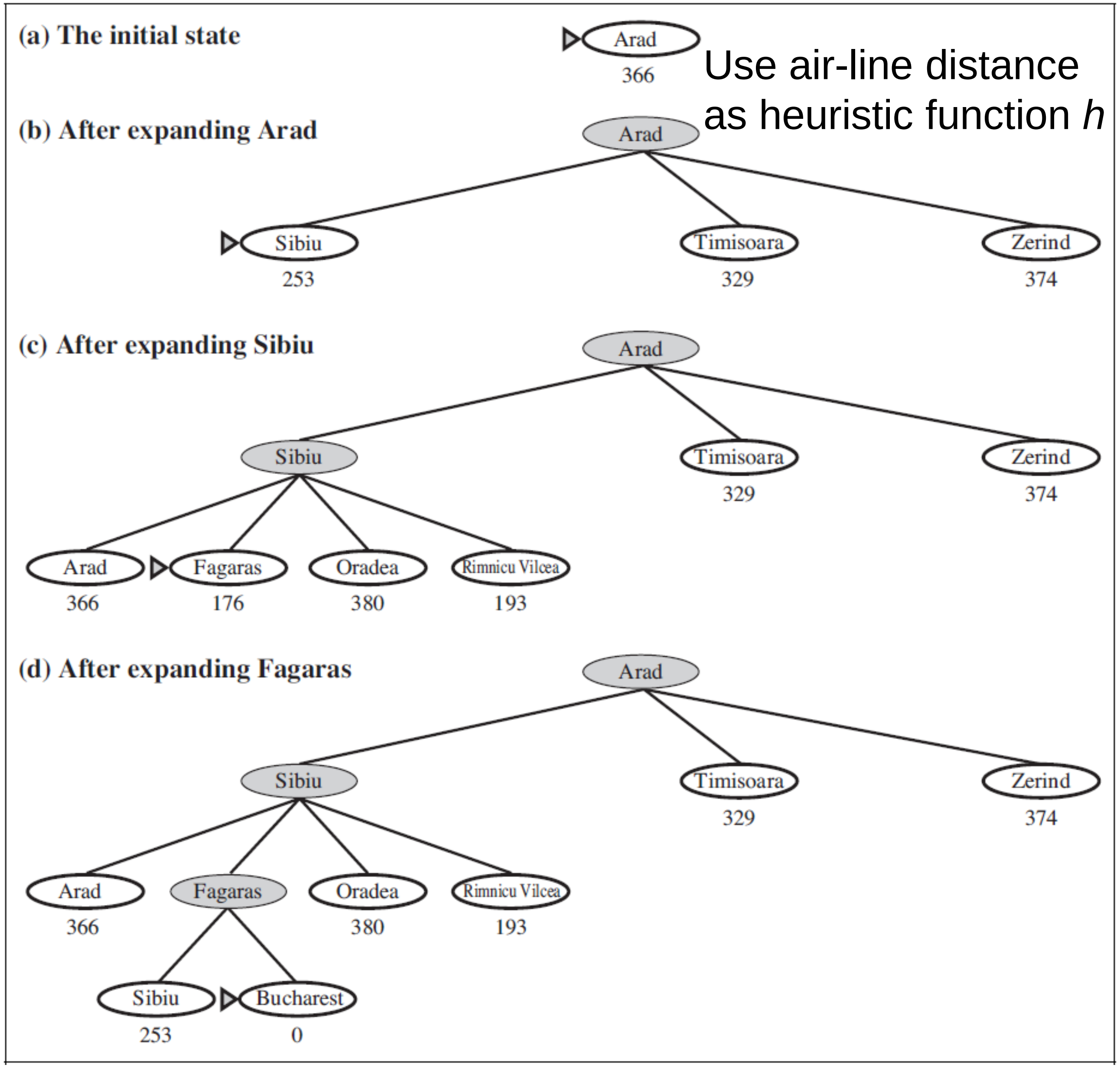


Use air-line distance as heuristic function *h*

Heuristics

- In case of greedy search, the evaluation function *h* is called a heuristic function or simply heuristic
- Name comes from greek ευρισκειν (to find, „Eureka!“)

- In AI:
  - Heuristics are fast, but probably incomplete methods for solving problems [Newell, Shaw, Simon 1963]
  - Heuristics are a means to accelerate search in average case

- A heuristic is problem-specific and focused on search

A* algorithm

- Minimizes the estimated path costs
- Combines uniform cost search and best first greedy

g(n): cost so far to reach *n*

h(n): estimated cost from *n* to a goal node

f(n) = g(n) + h(n): estimated total path cost through *n*

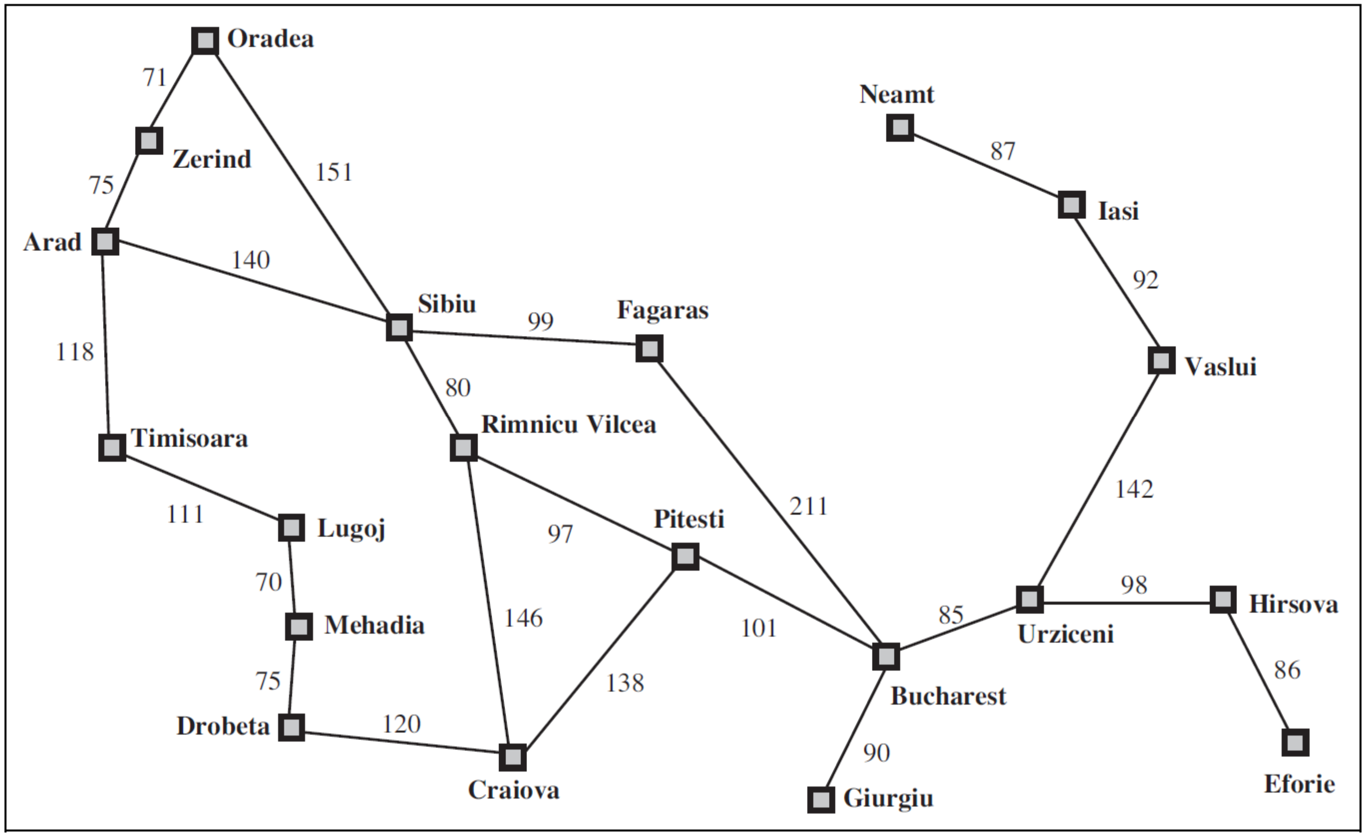
Let *h** be the true cost of an optimal path from *n* to goal

h is admissible, if for all nodes *n*:

$$h(n) \leq h^*(n)$$

h is optimistic, *h* never overestimates the actual costs

A*: From Arad to Bucharest



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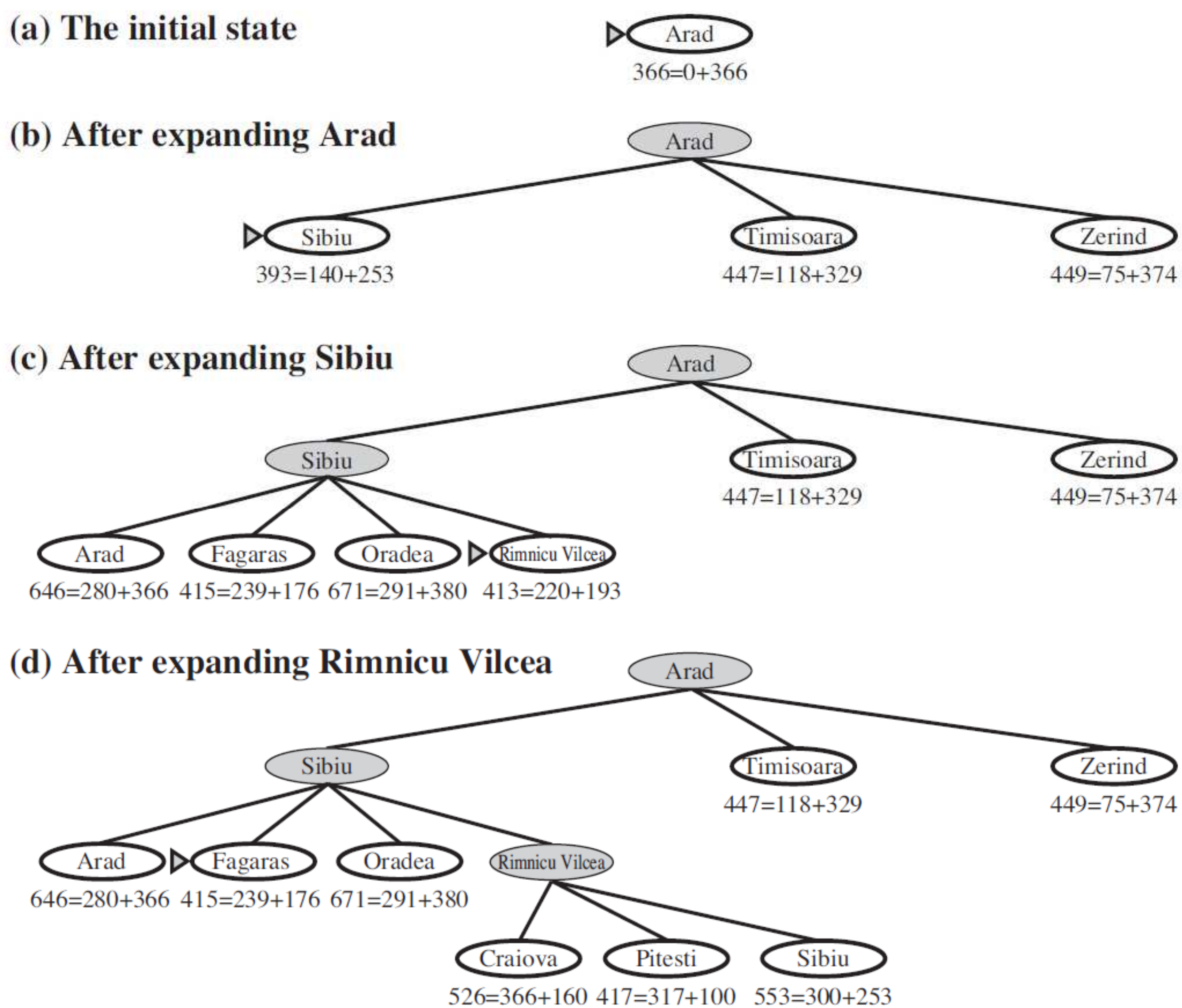
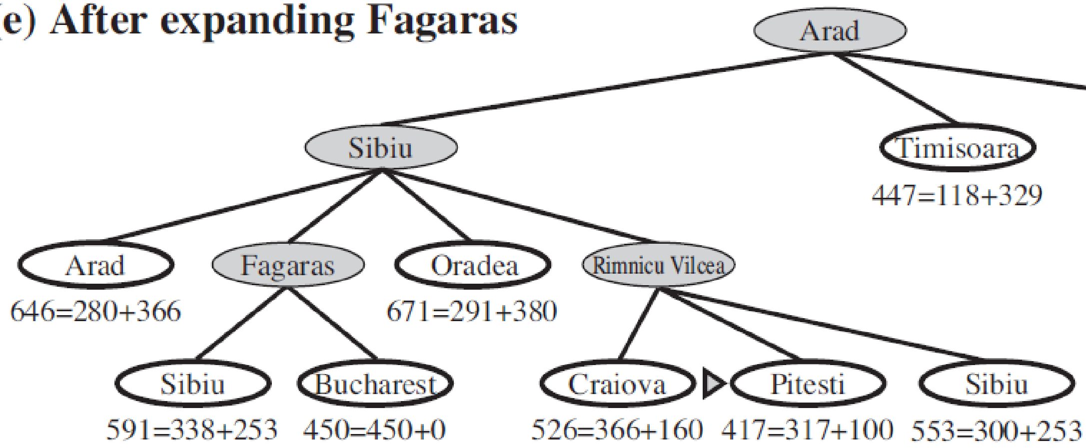
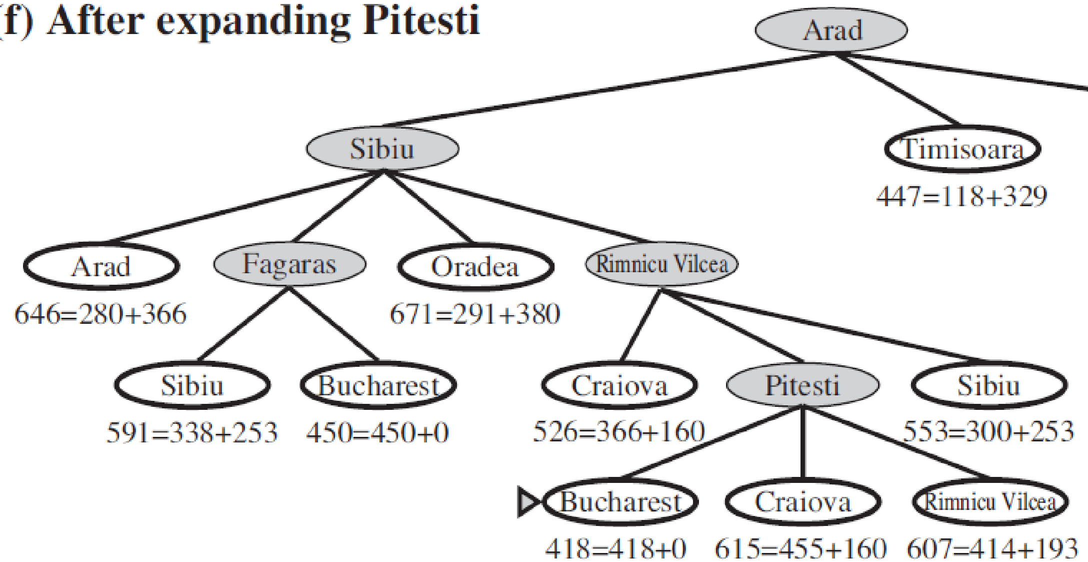


A*: From Arad to Bucharest

(e) After expanding Fagaras



(f) After expanding Pitesti



A* algorithm: properties

h is admissible, if for all nodes *n*: $h(n) \leq h^*(n)$

A (slightly) more strict condition:

Consistency (monotony):

h is consistent, if for all nodes *n*:

$$h(n) \leq c(n,a,n') + h(n')$$

where $c(n,a,n')$ are the costs from node *n* to a successor node *n'* as a result of the action *a*

Thesis: If *h* is consistent, then *h* is also admissible

A* algorithm: properties

Two versions of A*:

- Tree-search based
- Graph-search based

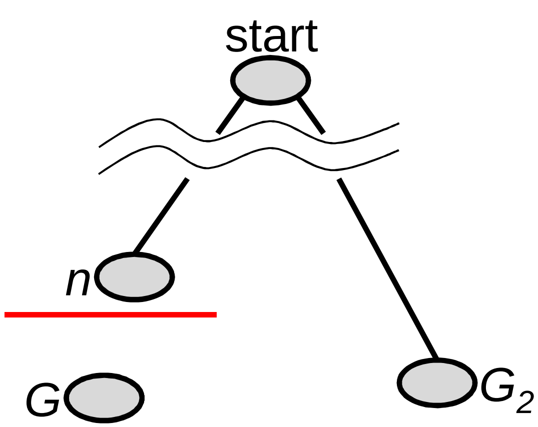
Theorem: A* is optimal if

- *h* is admissible in case of tree-search based A*
- *h* is consistent in case of graph-search based A*

A* algorithm: Optimality of tree-search form

Thesis: A* is optimal, i.e. the first solution found by A* has minimal costs

Proof: Assume there exists a goal node G with optimal path costs f^*, but A* has found a different goal G_2 with $g(G_2) > f^*$



A* algorithm: Optimality of tree-search form

Let n be a node on the optimal path from *start* to G which has not been expanded. Since h is admissible,

$$f(n) \leq f^* .$$

But because n hasn't been expanded before G_2, it holds that

$$f(G_2) \leq f(n)$$

From this it follows that

$$f(G_2) \leq f^*.$$

Because $h(G_2) = 0$ by definition, it follows that

$$g(G_2) \leq f^*.$$

⚡ to assumption $g(G_2) > f^*$. *Proof by contradiction.*

A* algorithm: Optimality of graph-search form

If h is consistent, the values of $f=g+h$ are monotonically increasing (not strictly).

Let n' be a successor node of n. For an action a holds

$$g(n') = g(n) + c(n,a,n')$$

This leads to

$$f(n') = g(n')+h(n') = g(n) + c(n,a,n') + h(n') \geq g(n)+h(n) = f(n)$$

$\geq$

A* algorithm: Optimality of graph-search form

If *h* is consistent, the values of $f=g+h$ are monotonically increasing (not strictly).

Let *n'* be a successor node of *n*. For an action *a* holds

$$g(n') = g(n) + c(n,a,n')$$

This leads to

$$f(n') = g(n')+h(n') = g(n) + c(n,a,n') + h(n') \geq g(n)+h(n) = f(n)$$

$$\geq$$

Now to prove: If a node *n* was chosen for expansion, then the optimal path to *n* has been found

A* algorithm: Optimality of graph-search form

Assume there is another cheaper path from *start* to *n.*

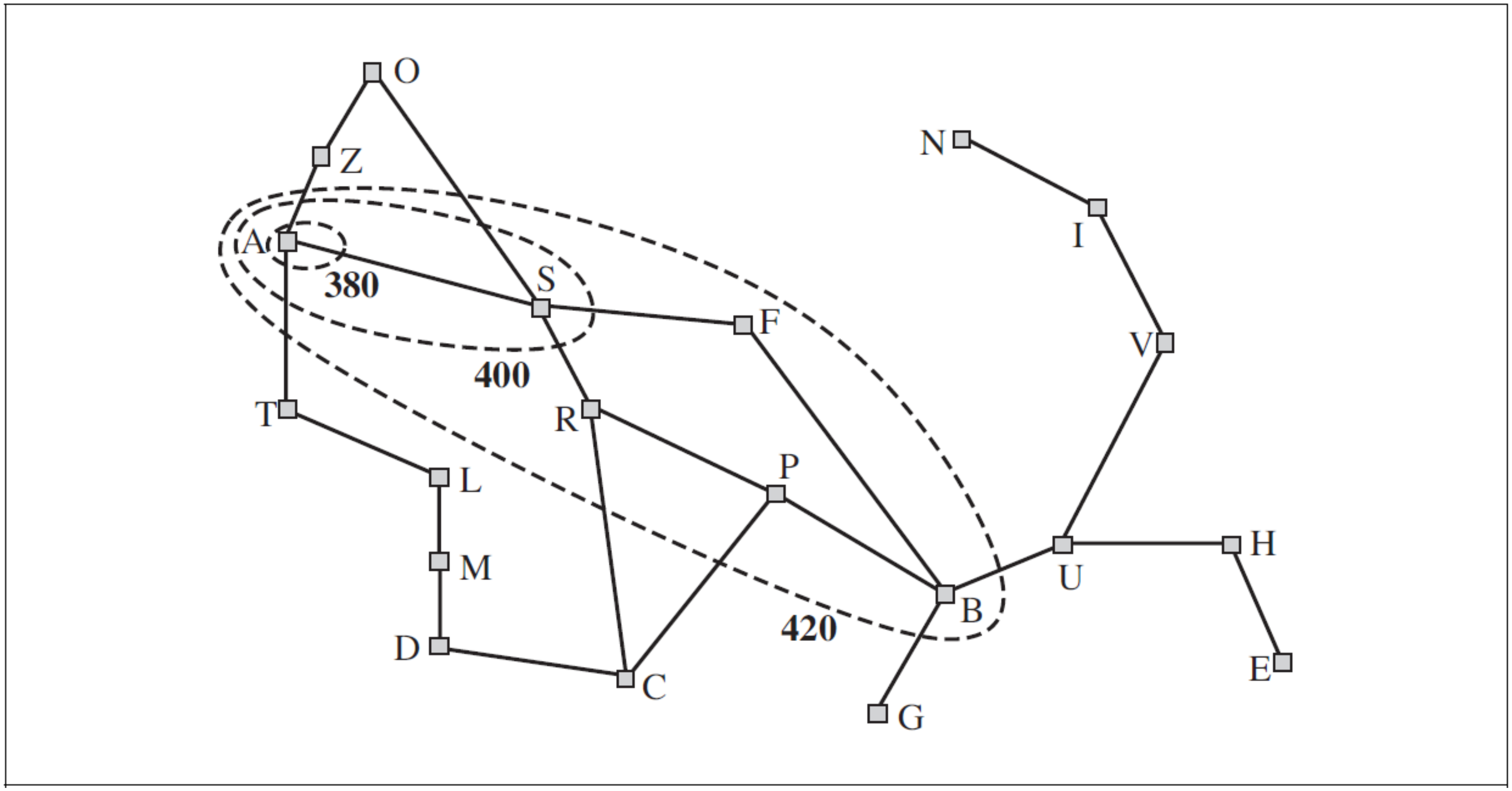
Then there is a node *n'* on that path with *f(n') < f(n)* because of monotony of *f* along any path.

Contradiction to algorithm definiton: n' would have been chosen instead of another node in the same set of frontier nodes because its costs are lower.

Then, taking *h*(goal)=0 into account, the function *f* gives the true cost for any goal and the costs for all other nodes on the way are at least as expensive.

A* algorithm: Optimality of graph-search form

We can draw a “contour map“ with nodes within a f-cost limit



A* algorithm: Properties

- A* expands all nodes with $f(n) < C^*$
  - C^* are the costs of an optimal path
- Completeness requires that there is only a finite number of nodes with with $f(n) < C^*$
  - True, if step costs $> \varepsilon > 0$ and branching factor b is finite
- No node with $f(n) > C^*$ is expanded
- If not all nodes with $f(n) < C^*$ are expanded, an algorithms risks to miss the optimal solution

A* algorithm: Properties

- A* is complete
- A* is optimal
- But: Number of configurations still exponential, even with pruning!
- Time exponential, but drastically reduced
- Space is the major problem

- Variation of A*: IDA* (Iterative deepening A*)
  - Pruning based on f-costs (g+h) instead of d
  - Because of iteration: no need to keep track of priority queue

Summary

- There are optimal and complete search algorithms which are “much better” than blind search
- However, the state spaces and the complexity is still exponential

- A* always leads to optimal solutions, but space is a problem.
  - Variations of A* to save space

Questions:

Restriction of costs to positive values:

a) Why would an optimal algorithm need to expand the whole space in case of arbitrary negative costs?

b) Does a restriction to c(n,a,n‘) > min (negative val.) help?

- In case of trees and in case of graphs?

c) Assume there are loops and the world state is the same after a finite number of actions. What is the optimal strategy in case of negative path costs for all actions?

d) Are there negative costs in real life?

Questions:

True or false?

a) Depth-first expands always at least as many nodes as A* with an admissible hueristic

b) For the 8-puzzle, h(n) =0 is admissible.

c) A* is not suitable for robotics, because percepts, actions, and states deal with contiuous values.

d) In chess, a rook (Turm) can move only horizontally or vertically, but not jump over other chessmen. The manhatten distance is admissible for a move from A zu B

Questions:

In graph-based A*, there can be state spaces with suboptimal solutions if *h* is admissible, but not consistent. Show an example.