

# Vorlesung

## Grundlagen der Künstlichen Intelligenz

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## Chapter 3

# Solving Problems by Searching: Informed (Heuristic) Search (cont'd)

# Finding heuristic functions

- What is a good heuristic function?

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

- $h_1$  = number of tiles at wrong location
- $h_2$  = sum of distances between tiles and their goal location (Manhattan distance)



# Empirical evaluation of different heuristics

- $d$  = distance to goal
- Average over 100 instances

d	Search Cost			Effective Branching Factor		
	IDS	$A^*(h_1)$	$A^*(h_2)$	IDS	$A^*(h_1)$	$A^*(h_2)$
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	364404	227	73	2.78	1.42	1.24
14	3473941	539	113	2.83	1.44	1.23
16	—	1301	211	—	1.45	1.25
18	—	3056	363	—	1.46	1.26
20	—	7276	676	—	1.47	1.27
22	—	18094	1219	—	1.48	1.28
24	—	39135	1641	—	1.48	1.26



## Effect of heuristic precision

Effective branching factor: Let  $N$  = number of expanded nodes

- $d$  = depth of solution in search space
- then  $b^*$  is the branching factor of the uniform search tree with depth  $d$  and  $N$  nodes
- $N+1 = 1 + b^* + (b^*)^2 + \dots + (b^*)^d$

Dominance of heuristics

- $h_1$  dominate  $h_2$ , if for all nodes  $n$  is true that:  
$$h_1(n) \geq h_2(n)$$
- This also means that  $A^*$  with  $h_1$  expands less nodes than  $h_2$  on average



# Choice of heuristics

- If possible, choose heuristics with higher values
  - Needs to be admissible/consistent
  - Check for calculation time of heuristics
- Example:  $h_1$  and  $h_2$  are heuristics for the 8-puzzle
- They also describe the **exact** path length for **relaxed problems**
  - Relaxed problem solved by  $h_1$  : Arbitrary jump of each field to the empty one
  - Relaxed problem solved by  $h_2$  : Any move (one step horizontally or vertically) is possible, even if position occupied



## Choice of heuristics

- What if there is no “unambiguously best“ heuristic?
- Assume, several (admissible/consistent) heuristics  $h_1, h_2, \dots, h_m$  exist. How to choose?

- Combine all!

$$h(n) = \max (h_1(n), h_2(n), \dots, h_m(n))$$

This takes the most precise one for each node.

- Given that  $h_1, h_2, \dots, h_m$  are admissible/consistent. Does this also hold true for  $h$ ?

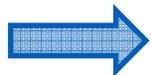


## Chapter 4

# Beyond Classical Search

# Local search and optimization

- Up to now:
  - systematic exploration of search spaces
  - Keep track of alternatives for each node along the path
  - The path is the solution
- What if only the **final state** is of interest for the solution?

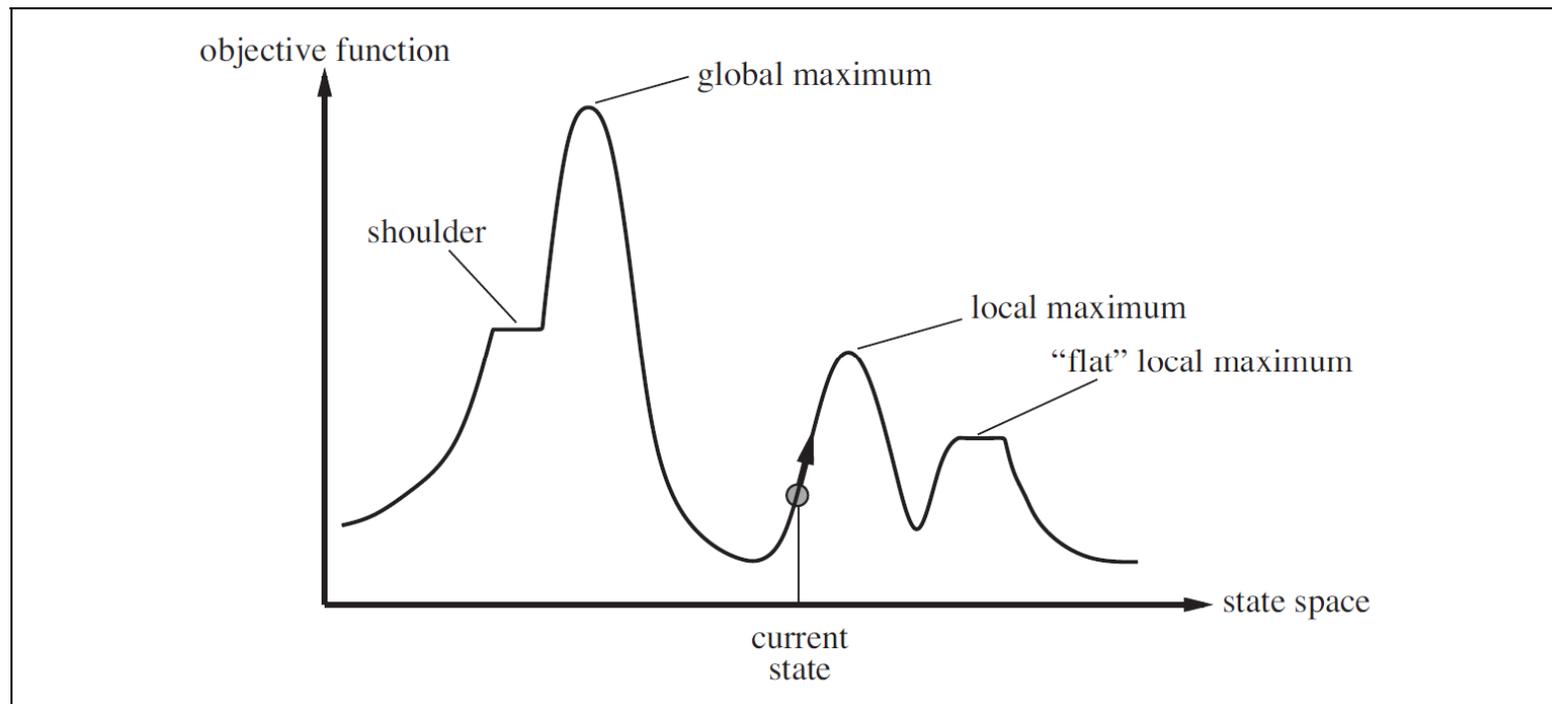
 Local search

- Examples:
  - 8-queens problem
  - VLSI design,
  - TSP



# Local search and optimization

- Define an objective function that evaluates states
- Use this function to optimize the search for a solution
- Idea: start with a random configuration and increase the solution stepwise ➡ Hill climbing



# Hill climbing

- Define an objective function that evaluates states
- Goal: maximizing the objective function

```
function HILL-CLIMBING(problem)  
returns a state that is a local maximum  
inputs: problem, a problem  
local variables: current, a node  
                  neighbor, a node  
  
current ← MAKE-NODE(problem.INITIAL-STATE)  
loop do  
    neighbor ← a highest-valued successor of current  
    if neighbor.VALUE ≤ current.VALUE then  
        return current.STATE  
    current ← neighbor  
end
```



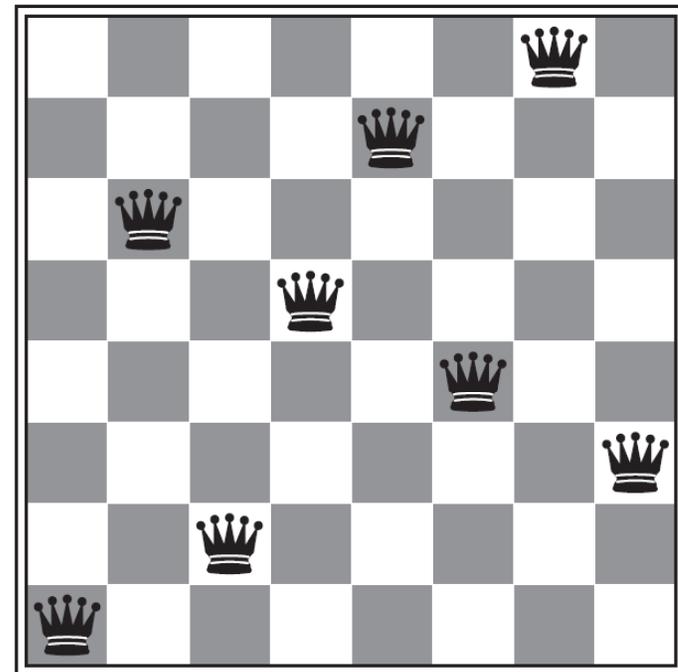
# Hill climbing: Example 8-queen problem

- Cost function: number of attacks
- Next state: Only one vertical move (queens remain in column)

18	12	14	13	13	12	14	14
14	16	13	15	12	14	12	16
14	12	18	13	15	12	14	14
15	14	14	♔	13	16	13	16
♔	14	17	15	♔	14	16	16
17	♔	16	18	15	♔	15	♔
18	14	♔	15	15	14	♔	16
14	14	13	17	12	14	12	18

$h=17$

(a)



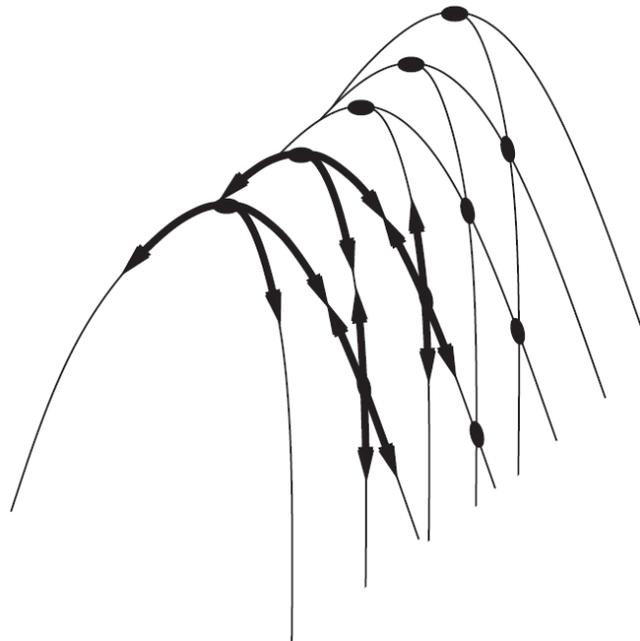
$h=1$  (local minimum)

(b)



# Problems of local search

- **Local maxima**: algorithm returns a sub-optimal solution.
- **Plateaus**: algorithm can only explore randomly.
- **Edges**: similar to plateaus.



# Problems of local search

- **Local maxima**: algorithm returns a sub-optimal solution.
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Solutions:

- **Re-start**, if no increase in performance
- **Noise**, random walk
- **Restricted search**: the last n operators cannot be applied

Strategies (and their parameters) that perform successfully (on a certain type of problem) can in most cases only be determined empirically.



# Simulated annealing

- Introduction of noise
- Imagine rough surface, “shake“ the system to overcome local minima

**function** SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state

**inputs:** *problem*, a problem

*schedule*, a mapping from time to “temperature”

**local variables:** *current*, a node

*next*, a node

*T*, a “temperature” controlling prob. of downward steps

*current* ← MAKE-NODE(INITIAL-STATE[*problem*])

**for** *t* ← 1 **to** ∞ **do**

*T* ← *schedule*[*t*]

**if** *T* = 0 **then return** *current*

*next* ← a randomly selected successor of *current*

$\Delta E$  ← VALUE[*next*] – VALUE[*current*]

**if**  $\Delta E > 0$  **then** *current* ← *next*

**else** *current* ← *next* only with probability  $e^{\Delta E / T}$



## Local beam search

- Restrict the nodes in memory to constant  $k$
- Initialize list with  $k$  random nodes
- Explore all successors of all  $k$  nodes
- Take the “best”  $k$  nodes out of this list, according to optimization function and use them for next step

Problem: concentration on small area (promising?) of the search space

- Updated list not with best  $k$  nodes, but with randomly chosen ones, based on a distribution given by the objective function



# Genetic algorithms

Evolution seems to be successful

**Idea:** Similar to evolution, solutions are searched by applying operators like "cross-over", "mutation" and "selection" to already successful solutions.

**Components:**

- **Encoding** of configurations as string or bit-string
- **"Fitness" function** that evaluates the goodness of a configuration
- **Populations** of configurations, initially random choice

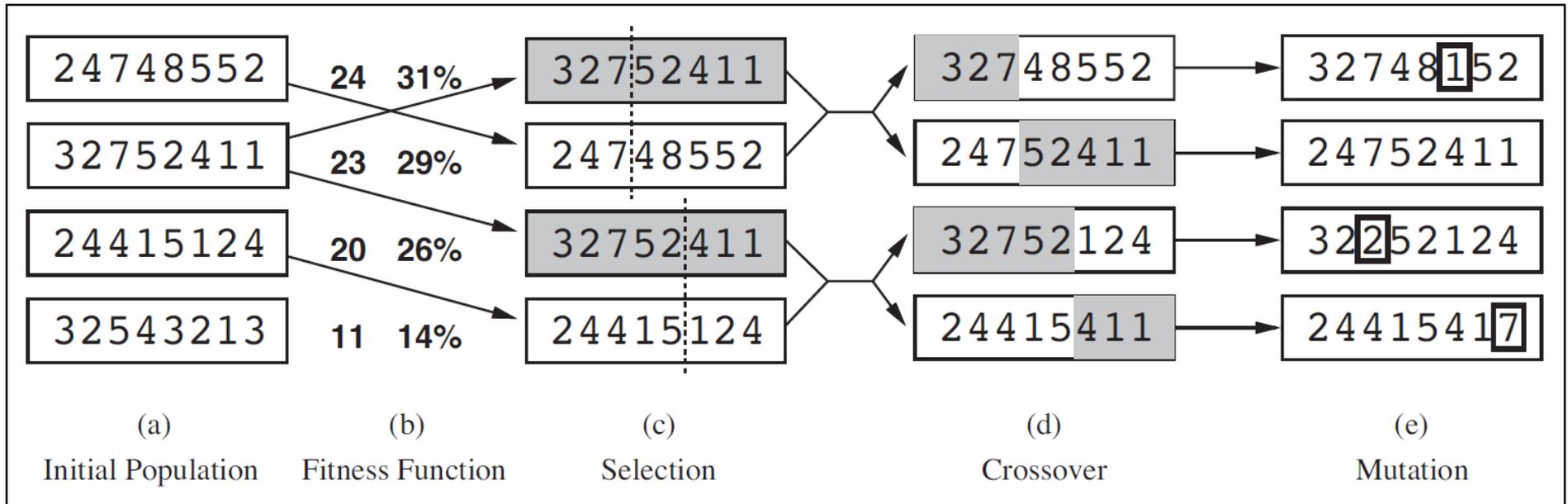
Example: 8 queens problem encoded as string of 8 digits.

Fitness function is computed based on the number of **non-attacks** ( $28=7+6+5+\dots+1$  for a solution)

Population consists of the set of queen configurations.



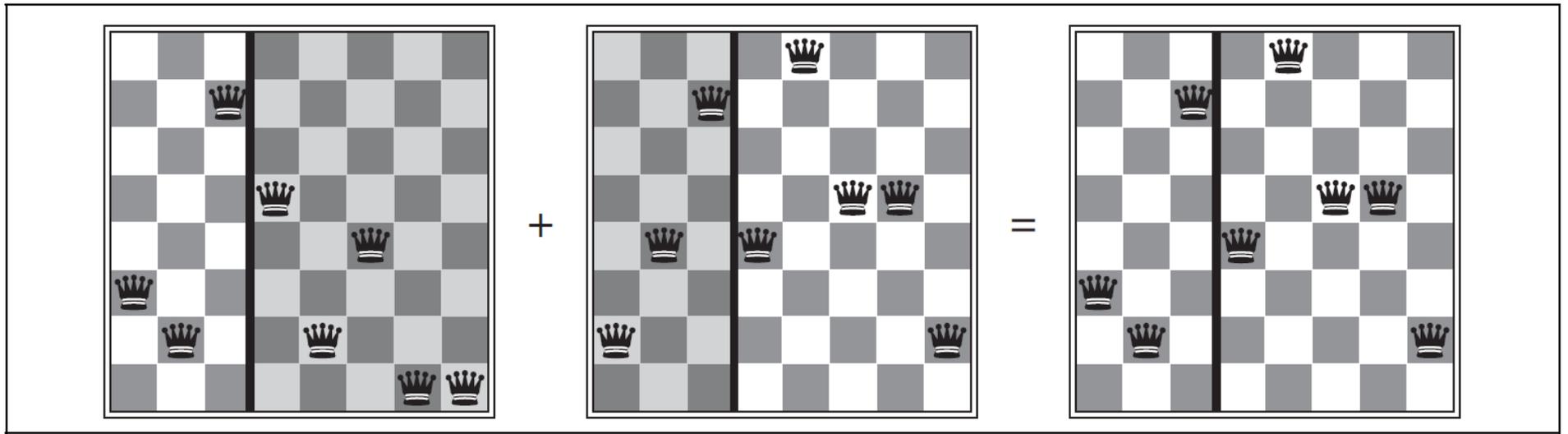
# Genetic algorithms: 8-queen problem



- Compute fitness for each configuration in population
- Choose two pairs for crossover, probability based on fitness
- Randomly choose crossover position for each pair
- Choose mutation with low probability

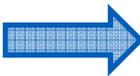


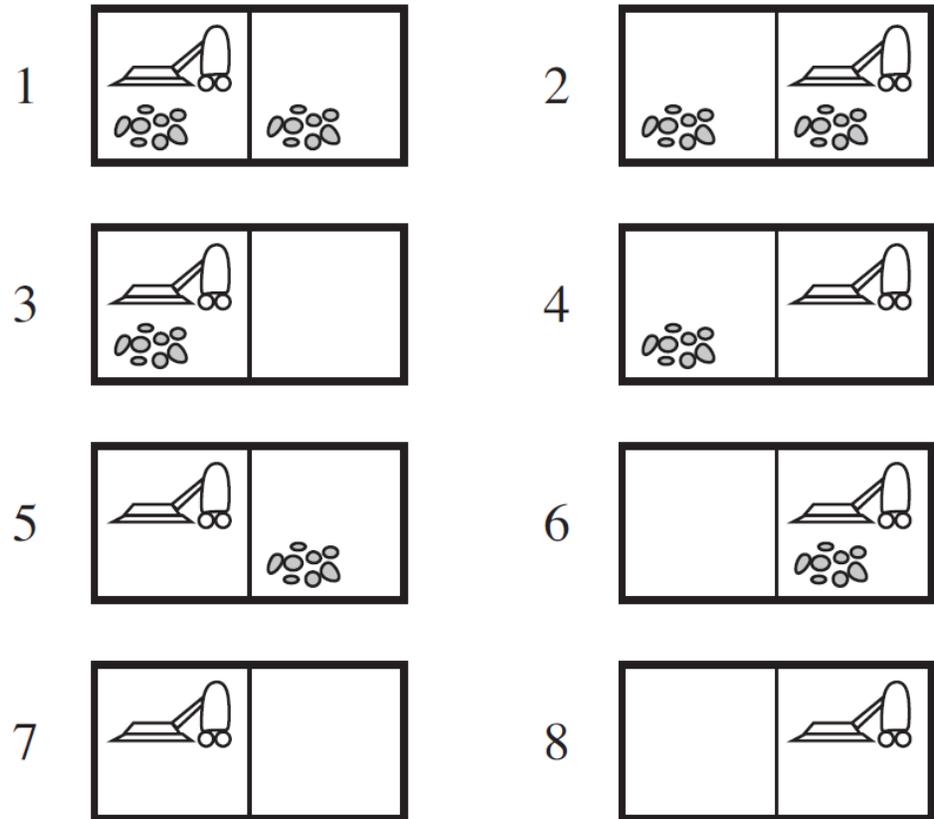
# Genetic algorithms: 8-queen problem



# Search with non-deterministic action results

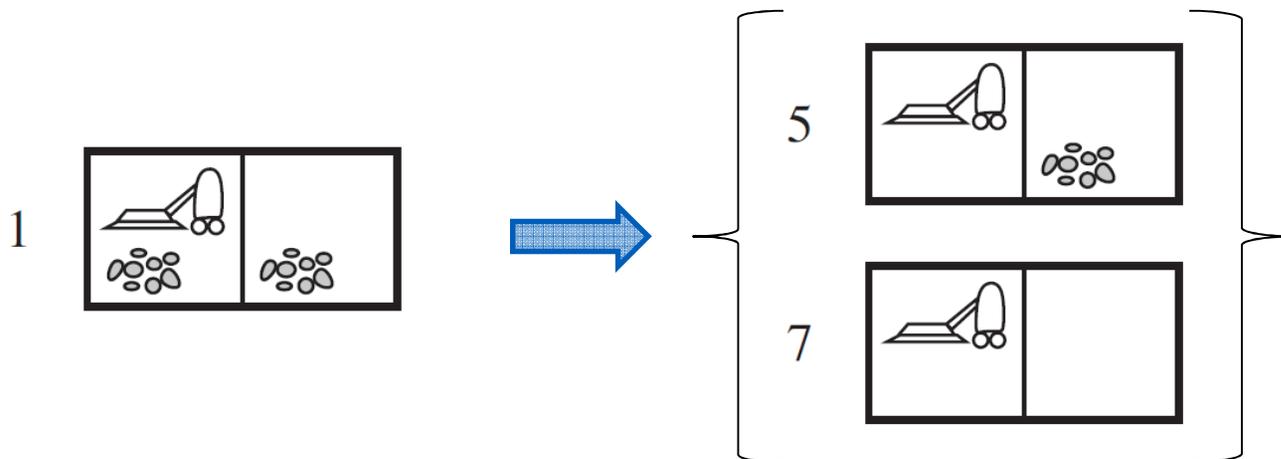
- Result of an action can be unobservable (or partially) observable
- Result of an action can be non-deterministic
- No clear sequence of actions possible

 contingency plan  
or strategy



# Search with non-deterministic action results

- Reconsider vacuum world with additional properties of the “suck” action:
  - Sometimes also the other field is cleaned
  - In case of a clean field, dirt may be released
- No unique result of an action, but a set of possible outcomes



# Search with non-deterministic action results

- Describe contingency plan in form of result-dependent action sequence
  - [action, result-dependent successor actions]
- Example:
  - [SUCK, **if** state=5 **then** [RIGHT, SUCK] **else** []]
- These resulting if-then-else cascades lead to decision trees
- Two types of branching out possible
  - Agent's own decision (what is the next action?)
  - Depending of the (non-deterministic) outcome of an action

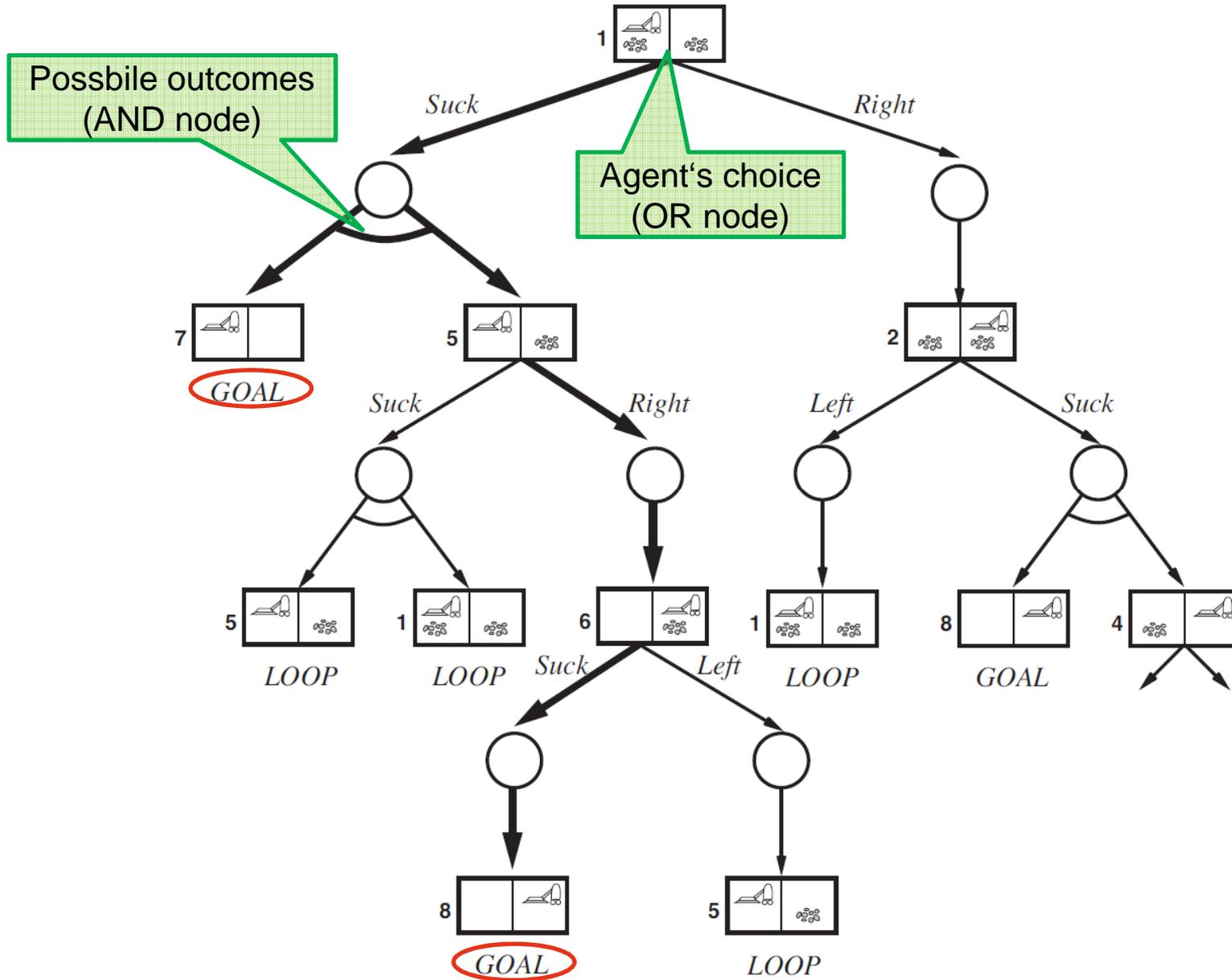


# Search with non-deterministic action results

- Search trees can be described as tree with two types of nodes
  - OR-nodes describe actions chosen by the agent
  - AND-nodes describe possible outcomes
- Alternating “layers“ of nodes (OR,AND) in the search tree
- A solution to a problem is a subtree with
  - A goal node at each leaf
  - An action for each OR node
  - All branches of an AND-node included
- Several search strategies can be applied, e.g. depth-frist, ...
- Finding heuristic functions is more complicated
  - Estimation of costs for a contingency plan instead of an action sequence

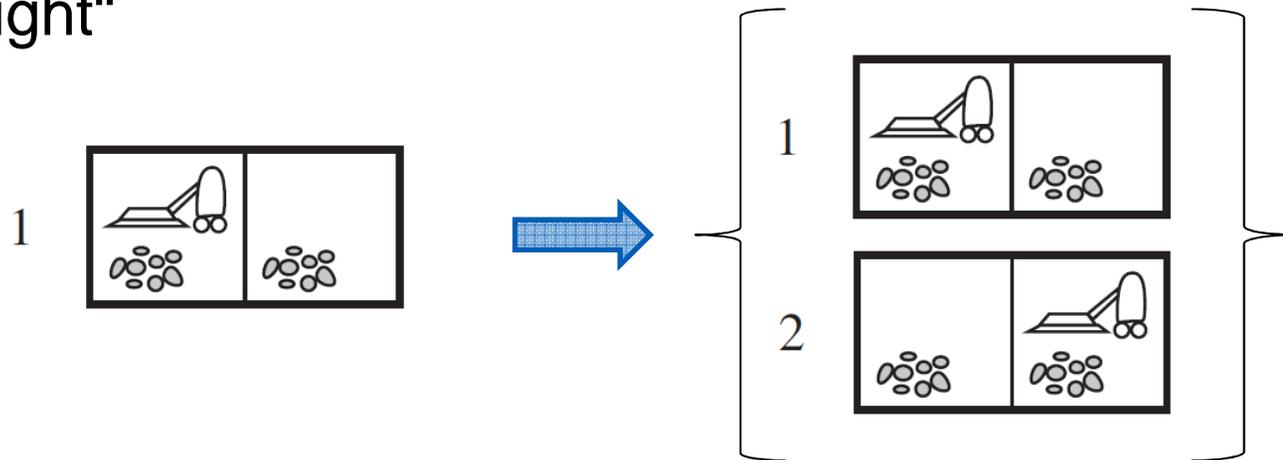


# Search with non-deterministic action results



# Search with non-deterministic action results

- What if “move” actions fail?  
E.g. “Right”

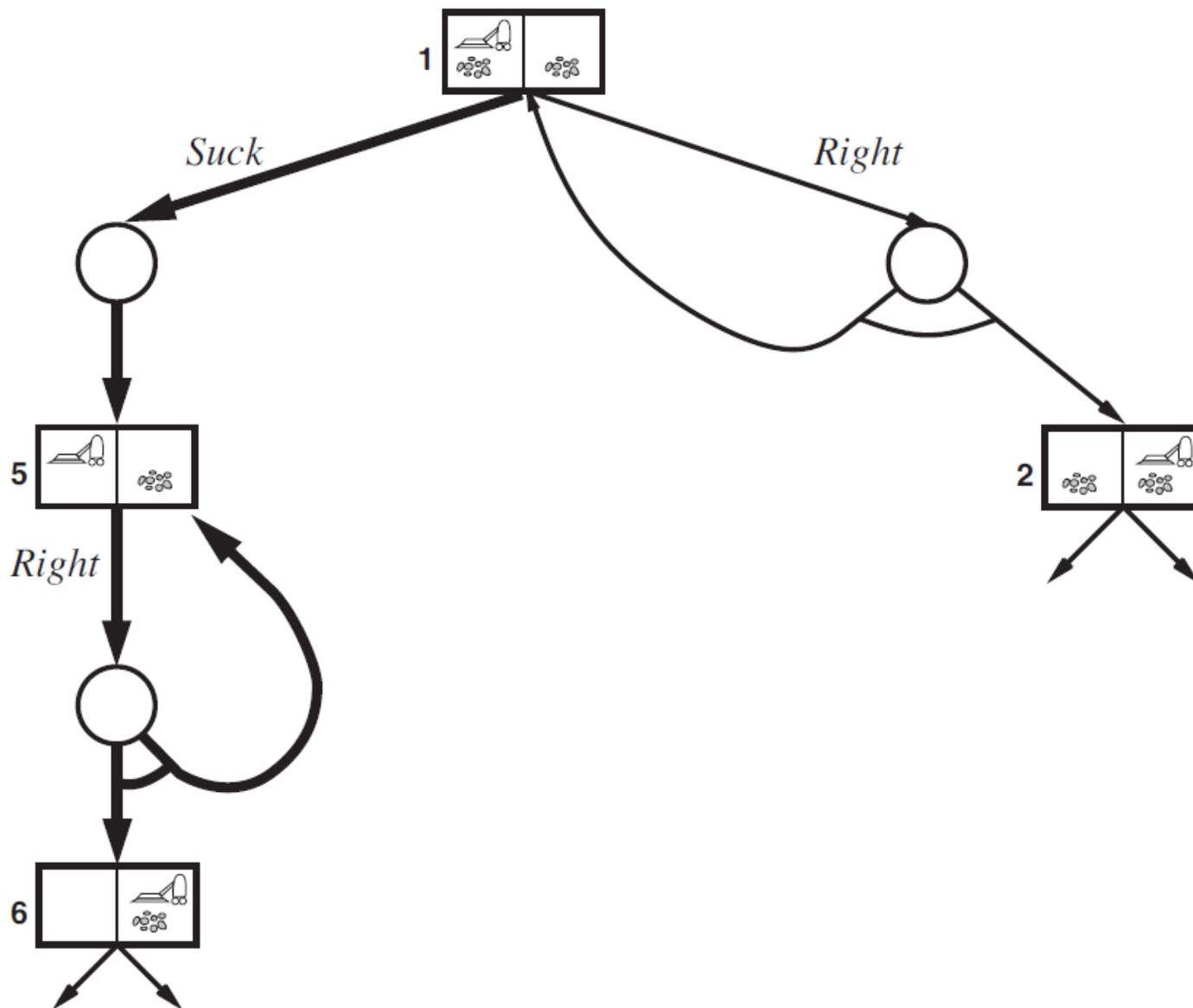


- No acyclic solution anymore, search fails
- Introduce labels for parts of plans  
[SUCK,  $L_1$  : RIGHT, **if state=5 then  $L_1$  else SUCK**]

or simply **while state=5 do right**



# Search with non-deterministic action results



# Summary

- Criteria for choosing “good“ heuristics
- Local search and optimization
  - Useful if only the final state is of interest
  - Problem: local minima, plateaus, etc.
  - Several algorithms: hill-climbing, simulated annealing, local beam search, genetic algorithms, etc.
- Search with non-deterministic action results
  - Contingency plan instead of action sequence
  - AND-OR-trees



**No class on Friday, 9<sup>th</sup> November 2012!**



When? 9-17h

Where? Immatrikulationshalle Campus Stadtmitte

