Technische Universität München
Department of Computer Sciene
I6 - Robotics & Embedded Systems
Dr. Reinhard Lafrenz
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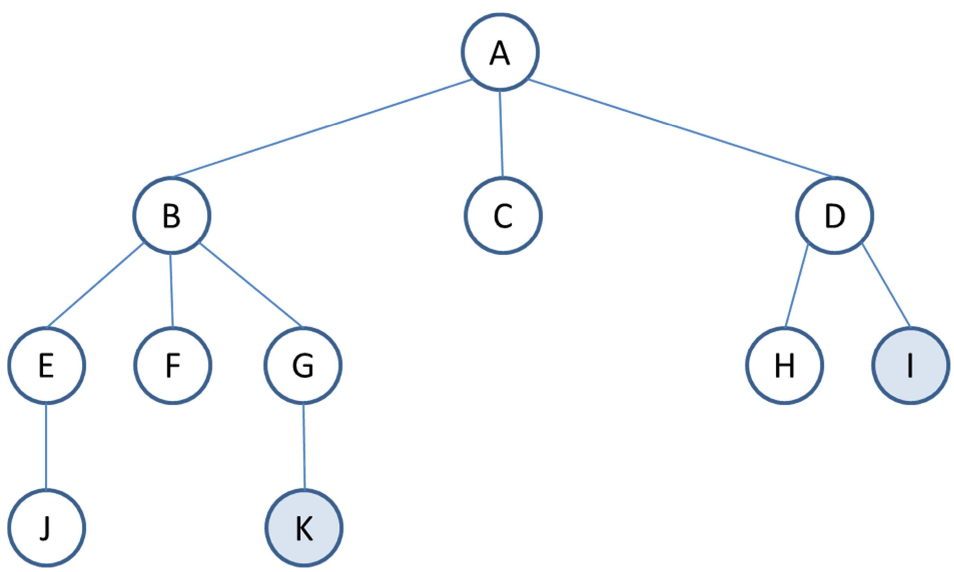
Exercises for the Lecture

Techniques in Artificial Intelligence

7.12.2012 - Sheet1

1) Search

Consider the following search graph



Assume that all direct connections between nodes have costs of 10 units. Start node is A, goal nodes are K and I.

a) Write down the nodes in order visited for different search strategies:

Depth-first search: A,B,E,J,F,G,K

Breadth-first search: A,B,C,D,E,F,G,H,I

Iterative deepening: A, A,B,C,D, A,B,E,F,G,C,H,I

b) Write down the nodes in the order visited for the A* algorithm using the following heuristic:

	A	B	C	D	E	F	G	H	I	J	K
h(.)	3	5	80	35	5	20	10	30	0	0	0

In case of equal values, the node to expand is chosen by **reverse** alphabetical order.

Give all intermediate steps and the values for g(n), h(n) and f(n). Explain why a node has been chosen.

Step 0. A is the only node in the Open list.

Step 1. Expand A.

Node	B	C	D
g(.)	10	10	10
h(.)	5	80	35
f(.)	15	90	45

Next node to expand is B, as it is the cheapest one with f=15

Step 2. Expand B.

Node	C	D	E	F	G
g(.)	10	10	20	20	20
h(.)	80	35	5	20	10
f(.)	90	45	25	40	30

Next node to expand is E, as it is the cheapest one with f=25

Step 3. Expand E.

Node	C	D	F	G	J
g(.)	10	10	20	20	30
h(.)	80	35	20	10	0
f(.)	90	45	40	30	30

Next node to expand is J with f=30. G has also f=30, but because of the choice by reverse alphabetical order, J is chosen.

Step 4. Expand J.

Node	C	D	F	G
g(.)	10	10	20	20
h(.)	80	35	20	10
f(.)	90	45	40	30

J is neither a goal node nor has successors, therefore next node to expand is G with f=30.

Step 5. Expand G.

Node	C	D	F	K
g(.)	10	10	20	30
h(.)	80	35	20	0
f(.)	90	45	40	30

Next node to expand is K with f=30. K is a goal node and f(K) is smaller than all other f values, therefore the algorithm terminates and returns K.

c) Is the solution found optimal? Explain.

No, I is a goal node with lower costs than K

d) Is the given heuristic admissible? If not, how can it be made admissible?

No. h(D)=35 overestimates the real costs to reach node I with g(I)=20. To make the heuristic admissible, change h(D) to a value <= 10, then the total cost estimation in node D is f(D)=g(D)+h(D)<=20.

e) Is the given heuristic consistent? Explain.

No. Being not admissible, the heuristic cannot be consistent.

f) Is the following statement true: "If h is an admissible heuristic, then h(n) = 0 for all goal nodes n". Explain.

Yes, goal states have no costs and the heuristic never overestimates.

2) Normal form in first-order logic

Transform the following sentences into the Conjunctive Normal Form (CNF) for first-order logic:

(a) $(\exists x(p(x,y))) \to (\exists x(q(x,x)))$

(b) $\forall x(\forall y \exists z(r(x,y,z)) \wedge \exists z \forall y(\neg r(x,y,z)))$

$$(\exists x(p(x,y))) \to (\exists x(q(x,x)))$$
$$=(\neg \exists x(p(x,y))) \vee (\exists x(q(x,x)))$$
$$=\forall x(\neg p(x,y)) \vee \exists x(q(x,x))$$

Rename variables

$$=\forall x(\neg p(x,y)) \vee \exists z(q(z,z))$$

Bring quantifiers to front

$$=\forall x \exists z((\neg p(x,y) \vee q(z,z))$$

Existential quantification of unbound variables

$$\exists y \forall x \exists z(\neg p(x,y) \vee q(z,z))$$

Skolemization of existentially quantified variables

$$\forall x(\neg p(x,a) \vee q(f(x),f(x)))$$

Write sentence in clause form

$$\{\{\neg p(x,a), q(f(x),f(x))\}\}$$

a)

$$\forall x(\forall y \exists z(r(x,y,z)) \wedge \exists z \forall y(\neg r(x,y,z)))$$

Rename variables

$$=\forall x(\forall y \exists z(r(x,y,z)) \wedge \exists z' \forall y'(\neg r(x,y',z')))$$

Bring quantifiers to front

$$=\forall x \forall y \exists z \exists z' \forall y'(r(x,y,z) \wedge \neg r(x,y',z'))$$

Skolemization of existentially quantified variables

$$\forall x \forall y \forall y'(r(x,y,f(x,y)) \wedge \neg r(x,y',g(x,y)))$$

Write sentence in clause form

$$\{\{r(x,y,f(x,y))\},\{\neg r(x,y',g(x,y))\}\}$$

b)

3) Resolution in First-Order Logic

Show that the following sets of clauses are **unsatisfiable** using resolution for first-order logic.

$$
\begin{aligned}
F = \{ & \{\neg p(y), q(x), r(x, f(x))\}, \\
& \{\neg p(y), q(x), s(f(x))\}, \\
& \{t(a)\}, \\
& \{p(a)\}, \\
& \{\neg r(a, y), t(y)\}, \\
& \{\neg t(x), \neg q(x)\}, \\
& \{\neg t(x), \neg s(x)\}\}
\end{aligned}
$$

$$F = \{\{p(x, a, x)\},$$
$$\{p(x, s(y), s(z)), \neg p(x, y, z)\},$$
$$\{\neg p(s(s(s(a))), s(s(a)), u)\}\}$$

(1) $\{\neg p(y), q(x), r(x, f(x))\}$

(2) $\{\neg p(y), q(x), s(f(x))\}$

(3) $\{t(a)\}$

(4) $\{p(a)\}$

(5) $\{\neg r(a, y), t(y)\}$

(6) $\{\neg t(x), \neg q(x)\}$

(7) $\{\neg t(x), \neg s(x)\}\}$

(8) $\{\neg q(a)\}$ (from (3) and (6) with $mgu = \{x|a\}$)

(9) $\{q(x), s(f(x)\}$ (from (2) and (4) with $mgu = \{y|a\}$)

(10) $\{s(f(a))\}$ (from (8) and (9) with $mgu = \{x|a\}$)

(11) $\{q(x), r(x, f(x))\}$ (from (1) and (4) with $mgu = \{y|a\}$)

(12) $\{r(a, f(a))\}$ (from (8) and (11) with $mgu = \{x|a\}$)

(13) $\{t(f(a))\}$ (from (5) and (12) with $mgu = \{y|f(a)\}$)

(14) $\{\neg s(f(a))\}$ (from (7) and (13) with $mgu = \{x|f(a)\}$)

(15) $\square$ (from (10) and (14))

(1) $\{p(x, a, x)\}$

(2) $\{p(x, s(y), s(z)), \neg p(x, y, z)\}$

(3) $\{\neg p(s(s(s(a))), s(s(a)), u)\}$

(4) $\{\neg p(s(s(s(a)))), s(a), z\}$ (from (2) and (3) with $mgu = \{x|s(s(s(a))), y|s(a), u|s(z)\}$

(5) $\{\neg p(s(s(s(a)))), a, z\}$ (from (2) and (4) Using renaming of variables $z|z'$ with $mgu = \{x|s(s(s(a))), y|a, z'|s(z)\}$

(6) $\Box$ (from (1) and (5) with $mgu = \{x|s(s(s(a))), z|s(s(s(a)))\}$

$$F = \{\{q(x), q(s(x))\},\ \{\neg q(x), \neg q(s(s(x)))\}\}$$

(1) $\{q(x), q(s(x))\}$

(2) $\{\neg q(x), \neg q(s(s(x)))\}$

(3) $\{q(s(x)), \neg q(x)\}$ (from (1) using renaming of variables $x|x'$ and (2) with $mgu = \{x'|s(x)\}$

(4) $\{q(s(x))\}$ (from (1) using renaming of variables $x|x'$ and (3) with $mgu = \{x|x'\}$

(5) $\{\neg q(x)\}$ (from (4) using renaming of variables $x|x'$ and (2) with $mgu = \{x'|s(x)\}$

(6) $\Box$ from (4) und (5) using renaming of variables $x|x'$ and with $mgu = \{x'|s(x)\}$