Becoming a Killer

A model of prey localisation and recognition by means of the lateral line

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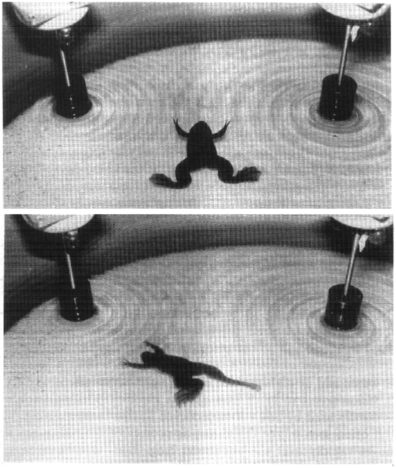
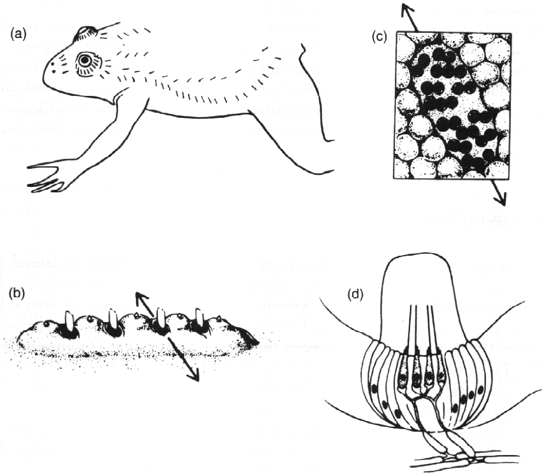
Xenopus laevis laevis

- completely aquatic
- carnivorous
- nocturnal
- lives in turbid waters
- prey localisation by means of the lateral line

Behavioural Experiments



Lateral Line System



Performance

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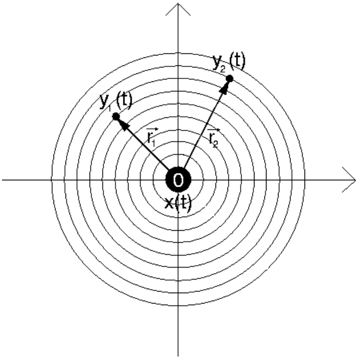
- wave source localisation ($\geq 5^\circ$ accuracy)
- frequency discrimination and recognition (approx. 4 % discrimination threshold)
- discrimination of the superposition of both waves
- normal operating range: $\leq 30\,\text{Hz}$

Water Waves I



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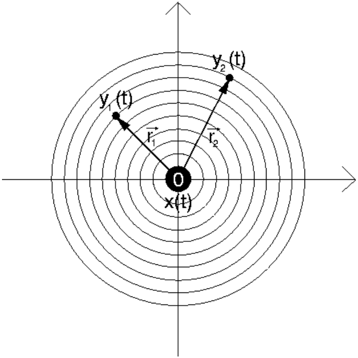

$$y_{1,2}\,(t) = \int_{-\infty}^{+\infty} h(t-\tau, \mathbf{r}_{1,2})x(\tau)d\tau \qquad (1)$$

Water Waves II

Fourier transformed with respect to time the equation looks like this

$$Y_{1,2}(\omega) = H(\omega, \mathbf{r}_{1,2})X(\omega) \ , \tag{2}$$

with the transfer function

$$H(\omega, \mathbf{r}) = \sqrt{\frac{r_0}{r}}\, 10^{-2|\Delta\varphi|/\pi} \times \exp\left[-\frac{4\nu k^3}{3\,\omega}(r - r_0) - kd - ik\,(r - r_0)\right] \tag{3}$$

$$\omega^2 = \left(gk + \frac{Tk^3}{\rho}\right)\tanh(Dk) \tag{4}$$

Impulse Response

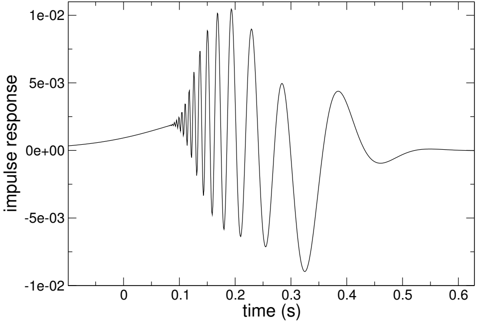

Figure: Impulse response at the position of an organ at 8 cm distance from the source.

Transfer Function

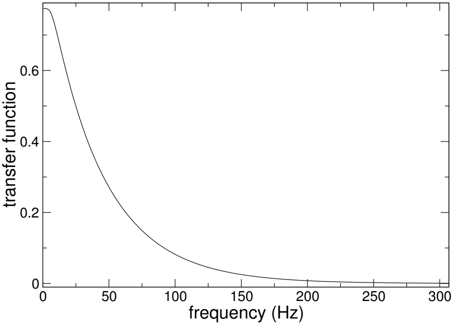

Figure: Same situation as on the last slide: Transfer function.

Outline of the Model

- stimulus localisation

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⇒ Construct a maximum-likelihood estimator for the original wave form!

Wave Form Reconstruction I

Create an estimator $\hat{x}(t)$ for the original wave form $x(t)$ by minimizing expectation of the error

$$\bar{E} = \left\langle \int [\hat{x}(t) - x(t)]^2 \, dt \right\rangle \quad , \tag{5}$$

where $\hat{x}$ can be written as

$$\hat{x} = \sum_j s_j \star y_j \; , \tag{6}$$

$$y_j = h_j \star x \; . \tag{7}$$

Wave Form Reconstruction II

Find the minimum of $\bar{E}$ with respect to the model parameters

$$\frac{\delta \bar{E}}{\delta s_j(\tau)} = 2\left\langle \int \left\{ \sum_i \left[s_i \star y_i \right](t) - x(t) \right\} y_j(t-\tau) dt \right\rangle \stackrel{!}{=} 0 \ , \quad (8)$$

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Assuming the noise to be Gaussian and with $\sigma := \frac{\sigma_n}{\sigma_x}$ this Eq. is solved by

$$S_j = \frac{H_j^*}{\sum_i |H_i|^2 + \sigma^2} \ . \quad (10)$$

Reverse Transfer Function I

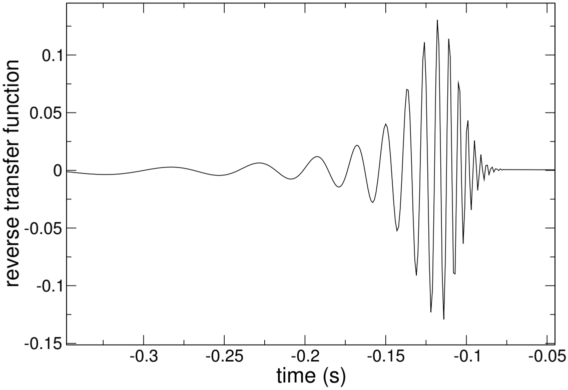

Figure: Reverse transfer function for an organ at 8 cm from the source (time domain).

Reverse Transfer Function II

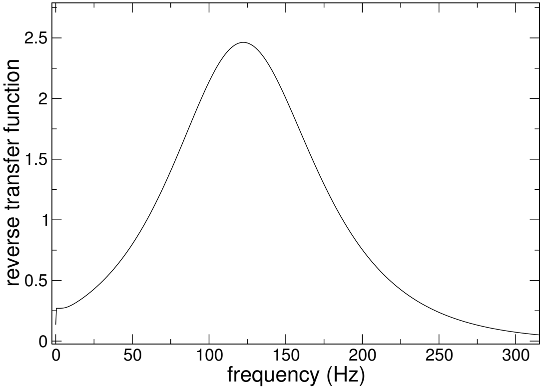

Figure: Same as the last slide: frequency domain.

Reconstruction of a Monofrequent Stimulus

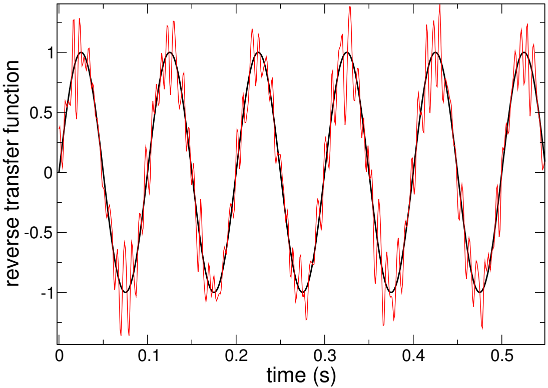

Figure: Reconstruction of the wave form using equation (7) ($f = 10\,\text{Hz}$, $\sigma_x = 1$, $\sigma_n = 0.1$).

Reconstruction of a Complex Stimulus

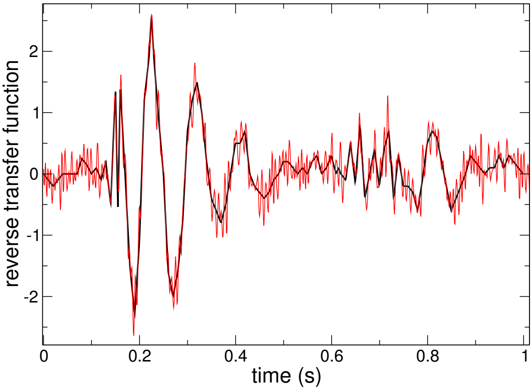

Figure: Same as on the last slide, but the stimulus is more realistic.

Localisation I

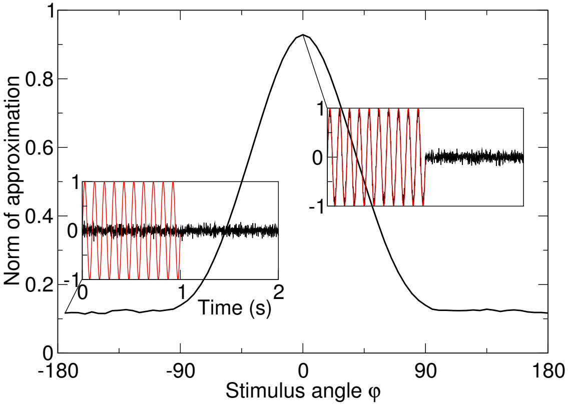

Figure: Map of reconstruction norms for various assumed source angles ($f = 10$ Hz, $\sigma_x = 0.5$, $\sigma_n = 0.1$)

Localisation II

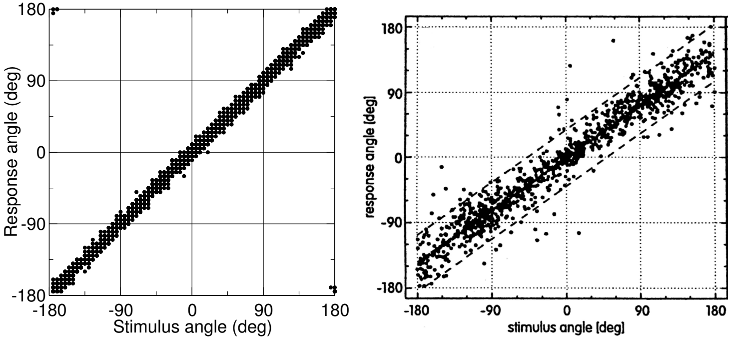

Figure: Localisation in model and experiment.

Wave Source Discrimination

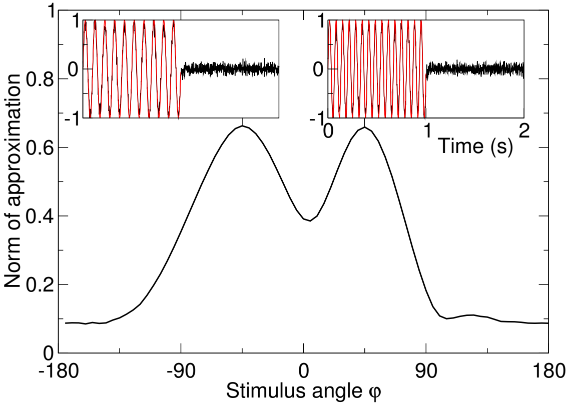

Figure: Two sources of 10 and 15 Hz, respectively, can be clearly discriminated.

Learning Algorithm

- return to the gerneralised gradient of the expected error (equ. (8))
- online learning rule to find the minimum: descent along the gradient

$$\Delta s_j(\tau) = -\eta \frac{\delta E}{\delta s_j(\tau)} \tag{11}$$

$$= \mathcal{F}^{-1}\left[-2\eta\left(\sum_i S_i Y_i - X\right) Y_j^*\right] \tag{12}$$

Results I

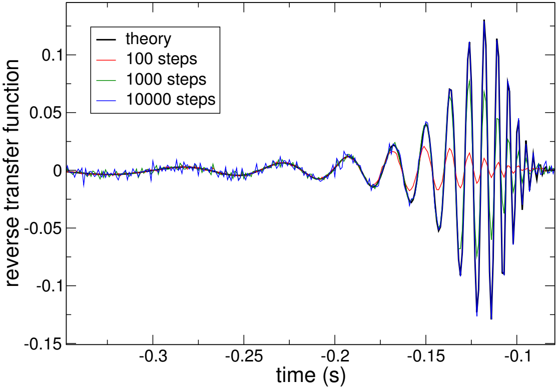

Figure: Linear learning: convergence ($\sigma = 0.1$, $\sigma_n = 0.1$, $\eta = 10^{-5}$).

Results II

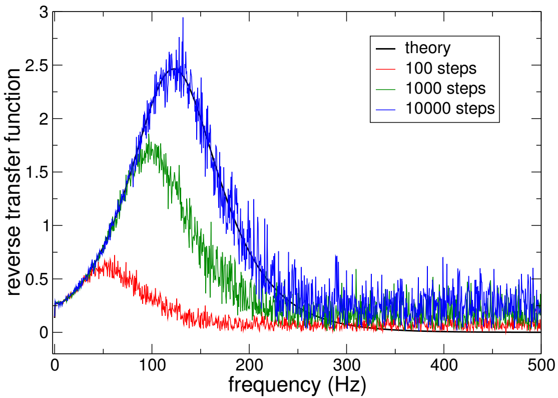

Figure: Convergence in frequency space.

Full-Field Theory I

We want to

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$\Rightarrow$ Find the maximum likelihood reconstruction with respect to the whole field!

$$0 \stackrel{!}{=} \frac{\delta \bar{E}_{\mathrm{ff}}}{\delta s_j^{\mathbf{p}}(\tau)} \tag{13}$$

$$\bar{E}_{\mathrm{ff}} = \left\langle \sum_{\mathbf{p}} \int_{-\infty}^{+\infty} \left[x^{\mathbf{p}}(t) - \hat{x}^{\mathbf{p}}(t)\right]^2 dt \right\rangle \tag{14}$$

Full-Field Theory II

It can be shown that $\hat{x}^{\mathbf{p}}$ is again found by convolution of the signal coming from the organs with reverse transfer fundtions

$$\hat{x}^{\mathbf{p}} = \sum_i s_i^{\mathbf{p}} \star y_i \ . \tag{15}$$

$\bar{E}_{ff}$ is minimised if $S_j^{\mathbf{p}}(\omega)$ solves the following system of linear equations:

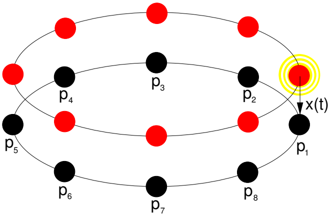
$$\sum_{\mathbf{p}} \left[\sum_i H_i^{\mathbf{q}*}(\omega) \, H_i^{\mathbf{p}}(\omega) + \sigma^2 \delta_{\mathbf{qp}} \right] S_j^{\mathbf{p}}(\omega) = H_j^{\mathbf{q}*}(\omega) \, . \tag{16}$$

Full-Field Learning

To learn this, we can use something like equation (12) again

$$\Delta s_j^{\mathbf{p}'}(\tau) = -\eta \frac{\delta E_{\mathrm{ff}}}{\delta s_j^{\mathbf{p}'}(\tau)} \quad (17)$$

$$= \mathcal{F}^{-1}\left[-2\eta\left(\sum_i S_i^{\mathbf{p}'} Y_i - \delta_{\mathbf{p},\mathbf{p}'} X^{\mathbf{p}}\right) Y_j^*\right] \quad (18)$$



Full-Field Calculation

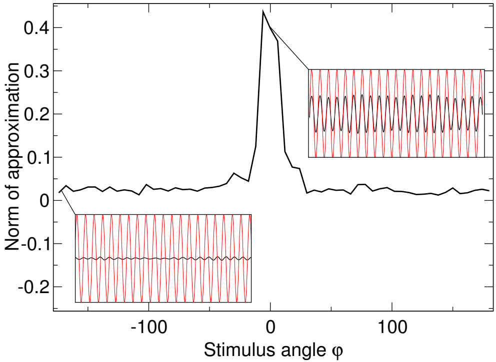


Figure: Map of reconstructions with a learned model (60 organs, 200000 steps).

Convergence

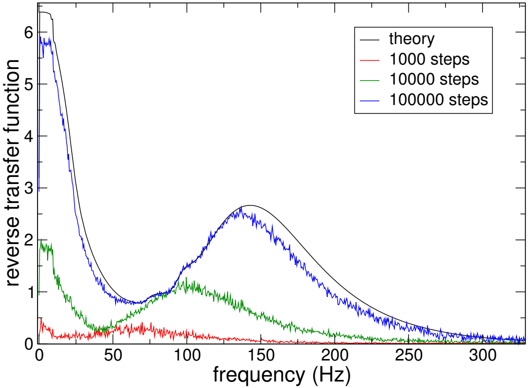


Figure: Full-field learning session (60 units).

Problems with Full-Field Learning

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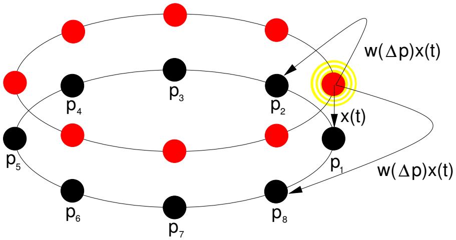
⇒ Find a compromise between single position and full-field reconstruction!

Window of Reference I

The idea is to give reference input $x^{\mathbf{p}}(t)$ to a unit assuming input at position $\mathbf{p}'$ weighted by the window of reference:

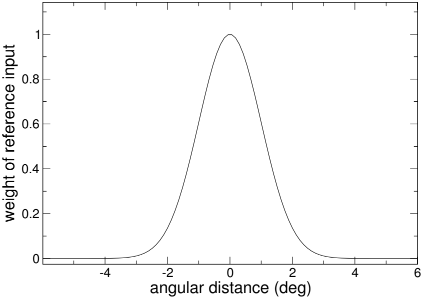
$$\Delta s_j^{\mathbf{p}'}(\tau) = \mathcal{F}^{-1}\left[-2\eta\left(\sum_i S_i^{\mathbf{p}'} Y_i - W(\Delta\mathbf{p})\, X^{\mathbf{p}}\right) Y_j^*\right] \qquad (19)$$



Window of Reference II

An example is be the gaussian function:

$$W\left(\mathbf{p}' - \mathbf{p}\right) = e^{-\frac{1}{2}\frac{\Delta\varphi^2}{\sigma_\varphi^2}} \qquad (20)$$



Learning a WOR model

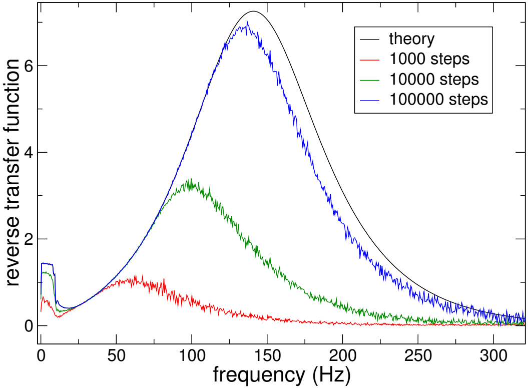


Figure: Convergence of a WOR model ($\sigma_\varphi = 12°$).

Performance of a WOR Model

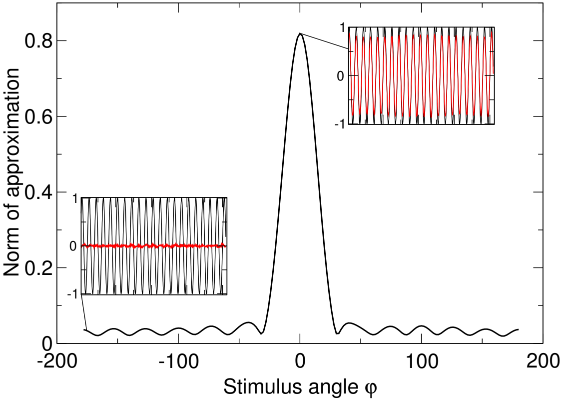
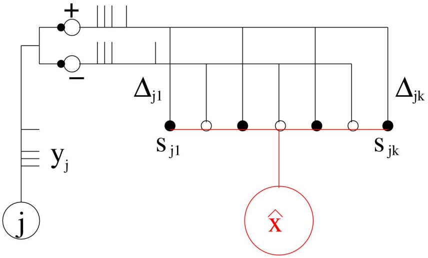


Figure: Localisation performance of a WOR model ($\sigma_\varphi = 10^\circ$).

Neuronal Implementation of a Convolution

$$\hat{x}(t-T) = \sum_{jk} s_{jk} y_i (t - \Delta_{jk}) \tag{21}$$



Basic Learning Equation

Synaptic learning with a learning window $W(t_2 - t_1)$:

$$\Delta J_{ik} = \eta \int dt_1 \int dt_2 W\left(t_2 - t_1\right) \Sigma^{\text{in}}_{ik}\left(t_2\right) \Sigma^{\text{out}}\left(t_1\right) \quad . \qquad (22)$$

Choose $W(t) = -2\,\epsilon(-t)$ and make sure that $\Sigma^{\text{out}} = \hat{x}(t) - x(t)$ by giving $x(t)$ as additional inhibitory input to ensure equivalence to our learning equation

$$\Delta s_{ik} = \mathcal{F}^{-1}\left[-2\eta\left(\hat{X} - X\right) Y_i^*\right] \quad . \qquad (23)$$

Reverse Transfer Functions I

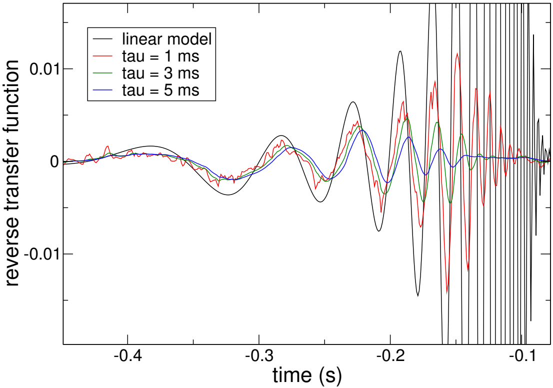


Figure: Reverse transfer functions implemented in neuronal hardware. The linear model can only provide rough comparison.

Reverse Transfer Functions I

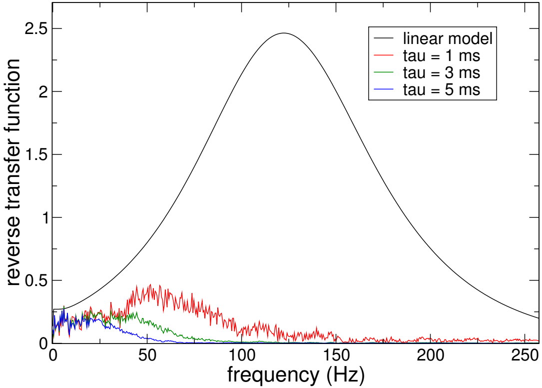


Figure: Same as the last slide in frequency space. The broadening of a spike in the PSPs acts as a low pass filter.

Localisation

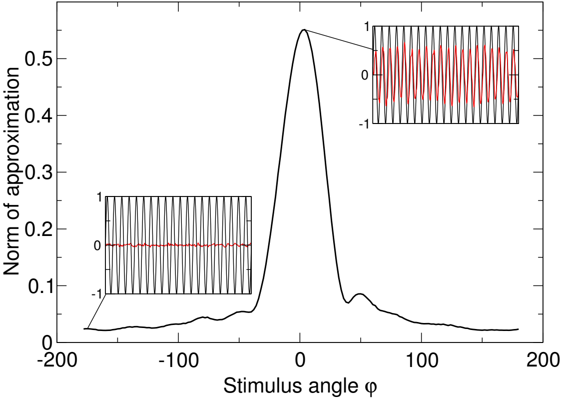


Figure: Localisation performance of a field of 180 neurons after 25000 learning steps.

Outlook

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- distance learning
- underwater detection
- reference input

Bibliography

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